Hydrofoil Craft Drag Polar

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Adaptation of the friction and wave drag components to the classic aerodynamic drag polar are shown with accommodation for the weight/center-of-gravity envelope. The parametric forms of the drag, power, and specific range and endurance curves are shown and related to the traditional dimensional forms. The relationship between the drag polar and the propulsion is indicated.

Nomenclature							
\boldsymbol{A}	= aspect ratio or propeller disk area, m ²						
C_D	= drag coefficient, $D/qS = C_T A/MS$						
	$= (P.C.)P/(\rho/2)V^3MS$						
C_f	= friction (Schoenherr) coefficient						
C_L	= lift coefficient, L/qS						
C_{L_D}	= design lift coefficient, L_D/q_DS						
$C_{L_{\text{ref}}}$	= any reference lift coefficient						
C_f C_L C_{LD} $C_{L_{\mathrm{ref}}}$ $C_{L_{VD}}$ C_{sfc} C_T	= C_L at design speed, $(L/L_D)C_{L_D}$ = $sfc P^{1/3}$, $N/sN^{2/3}$						
$C_{\rm sfc}$	= thrust coefficient, T/qA						
C'	= chord m						
D	= drag, N, total for craft without subscript						
E_S	= specific endurance, s/N						
F_h	= depth Froude number, V/\sqrt{gh}						
$f(F_h)$	$=\exp\left(-2/F_h^2\right)/2F_h^2$						
g	= acceleration of gravity, m/s ²						
h	= foil depth, m						
K_{W}	$=h/cC_{D_{W}}/f(F_{h})$						
k_{sep}	$=C_{D_{\text{sep}}}/(C_L-C_{L_{\text{ref}}})^2$						
k_0, k_1, k_2	= drag polar coefficients:						
	$C_D = k_0 + k_1 C_L + k_2 C_L^2$						
L	= craft net weight, displacement less buoyancy, N						
L_D	= design net weight, that net weight for which $T_D = M_D D_D$ and $T_{TO} \ge M_{TO} D_{TO}$, N						
1	= distance between forward and aft foil centers of						
	pressure, m						
l_{1}, l_{2}	=distance from forward and aft foil centers of						
	pressure to c.g., m						
M	=T/D, a design specification at design speed and takeoff						
N	= number of powerplants, associated with $C_{\rm sfc}$ to						
	preserve identification of total installed power						
	with total craft drag						
\boldsymbol{P}	= shaft power, $DV/P.C.$, W						
P.C.	= propulsion coefficient, $\eta \eta_G$						
q	= dynamic pressure, $\rho V^2/2$, Pa						
R_{S}	= specific range, m/N						
S	=total foil area, m ²						
T	=thrust available, N						

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V	= speed, m/s
V_D	= design speed, that speed for which $T_D = M_D D_D$ for
_	L_D , m/s
δ	= circulation distribution factor for induced drag
ρ	=density, kg/m ³
η	= propeller efficiency
η_G	= gear efficiency
Subscript	s
D	= at design speed
i	=induced
max	= maximum
min	= minimum
\boldsymbol{P}	= parasite
qD/2	= at half-design dynamic pressure
sep	= separation
surf	= surface image
TO	= at takeoff speed
VD	= at design speed
VD/2	= at half-design speed
W	= wave
1, 2	= forward and aft

Introduction

YDROFOIL craft drag curves are traditionally derived as a sum of component drags. The practice reflects a marine tradition dominated by the displacement hull problem which is still heavily dependent upon empiricism. Hydrofoil craft flight performance, however, is essentially aerodynamic and amenable to the methods and advantages of the aerodynamic analytic tradition.

Aerodynamic tradition would derive hydrofoil craft drag as a sum of drag coefficients to produce the drag polar which provides a convenient nondimensional display of the most pertinent design features affecting craft performance. Coefficients for the drag polar are based largely upon empirical data and this paper is limited to consideration of craft with fully submerged hydrofoils designed for fully wetted flow. With the representation of the hydrofoil wave drag in drag polar form, the total drag for such craft can be expressed analytically with a precision which matches that of the laboratory.

Aerodynamic tradition is rich in applications of the drag polar but this single paper must define the hydrodynamic drag polar and its application in very general terms.

Drag Polar

For the present, the hydrofoil craft drag polar can be considered to be a quadratic in the craft lift coefficient:

$$C_D = k_0 + k_1 C_L + k_2 C_L^2 \tag{1}$$

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where the craft lift coefficient is the averaged forward and aft hydrofoil lift coefficient:

$$C_L = L/qS = (S_1/S)C_{L_1} + (S_2/S)C_{L_2}$$
 (2)

The frequent aerodynamic neglect of the linear term is not prudent for the hydrodynamic case where this term aids assessment of the drag penalty associated with cavitation constraints and provides improved incorporation of the foil wave drag speed dependency.

The coefficients of the drag polar are depth dependent. Those sets of coefficients representing the craft for takeoff and for cruise at some design depth are of particular interest. The discussion here is limited to consideration of the drag polar at design cruise depth.

The coefficients of the drag polar are also functions of Reynolds number and Froude number and these speed dependencies are incorporated as curve fits suited to the objective and the desired precision.

Drag Polar Adaptations

It is frequently convenient to express the drag polar in terms of the dynamic pressure ratio q_D/q and the design speed lift coefficient $C_{L_{VD}}$

$$C_D = k_0 + k_1 C_{L_{VD}} (q_D/q) + k_2 C_{L_{VD}}^2 (q_D/q)^2$$
 (3)

This form is particularly convenient to the incorporation of the friction and wave drag speed dependencies. Any speeddependent coefficient can be written in the form:

$$k_x = k_{0_x} + k_{I_x} C_L + k_{2_x} C_L^2$$
 (4)

where the coefficients are the solutions for three equations of the form of Eq. (3) written for any dynamic pressures, inside or outside the flight speed range. It is convenient, for example, to employ the design speed, half-design dynamic pressure, and half-design speed points where the coefficients of Eq. (4) become:

$$k_{\theta_x} = (8/3)k_{x_D} - 2k_{x_{0D/2}} + (1/3)k_{x_{VD/2}}$$
 (5a)

$$k_{I_x} = \left[-2k_{x_D} + (5/2)k_{x_{qD/2}} - (1/2)k_{x_{VD/2}} \right] / C_{L_{VD}}$$
 (5b)

$$k_{2_X} = \left[(1/3) k_{x_D} - (1/2) k_{x_{qD/2}} + (1/6) k_{x_{VD/2}} \right] / C_{L_{VD}}^2$$
 (5c)

It will be recognized that this procedure fits the speed dependent coefficient at three points on the craft speed scale for a fixed craft weight. The three freedoms available need not all be employed and their location on the speed scale is arbitrary. The extreme speed points do not limit the range of validity for the curve fit.

Friction Drag

Over a flight range of half the design speed the friction drag coefficient is adequately represented by a linear variation with the dynamic pressure or lift coefficient:

$$C_f = C_{f_D} - m + \frac{m}{C_{LVD}} C_L \tag{6}$$

where

$$m = \left(C_{f_{VD/2}} - C_{f_D}\right) / 3$$

The friction drag coefficient variation in the flight speed range is less than the parasite drag allowance and is of doubtful significance. The variation is employed in the illustrations of this paper and some illustrations include fixed friction drag coefficient results for comparison.

Parasite Drag

The parasite drag coefficient is the sum of the products of the minimum drag coefficients and their reference areas for each element of the configuration, normalized by the total foil area:

$$k_{\theta_P} = \frac{I}{S} \sum_{i} \left(C_{D_{\min}} S \right)_i = \frac{I}{S} \sum_{i} D_{\min_i} / q \tag{7}$$

Equation (6) appears as a factor in the foil, strut, and pod minimum drag coefficients. The air drag coefficient is reduced by the water/air density ratio.

Infinite Depth Drag Due to Lift

The incremental foil profile drag over minimum drag, the separation drag, is inherently of polar drag form:

$$C_{D_{\text{sep}}} = k_{\text{sep}} \left(C_L - C_{L_{\text{ref}}} \right)^2$$

$$= k_{\text{sep}} C_{L_{\text{ref}}}^2 - 2k_{\text{sep}} C_{L_{\text{ref}}} C_L + k_{\text{sep}} C_L^2$$
(8)

where k_{sep} and $C_{L_{\text{ref}}}$ are section characteristics. The classic aerodynamic induced drag is employed and appears in the drag polar as:

$$C_{D_i} = \frac{l+\delta}{\pi A} C_L^2 \tag{9}$$

Hydrodynamic Drag Due to Lift

The hydrodynamics of the vortex line of finite span in the vicinity of the free-surface and in infinitely deep water have been investigated by Wu¹ and Breslin.^{2,3} Reference should be made to the original works for the details and procedure required to determine the theoretical characteristics, since the expressions derived are rather complicated and do not lend themselves to simple formulation or evaluation.

In pursuit of a convenient explicit expression for the freesurface drag, the Gibbs & Cox Handbook distinguished the infinite Froude number result which presents the hydrofoil operating beneath a rigid surface:

$$C_{D_{\text{surf}}} = \left(C_{D_{\text{surf}}}/C_L^2\right)C_L^2 \tag{10}$$

The coefficient for this term is the Prandtl biplane factor, identified as $K_b - 1$ in Ref. 4 and as σ_i by Wilson who suggests a convenient expression for its value in Ref. 5.

Reference 4 makes the remainder of the free-surface drag proportional to the lift-line wave drag of Kochin⁶:

$$C_{DW} = K_W \frac{1}{h/c} \left[\exp\left(-2/F_h^2\right) \middle/ 2F_h^2 \right] C_L^2$$

$$\frac{h/c}{K_W} C_{DW} = \left[\exp\left(-2/F_h^2\right) \middle/ 2F_h^2 \right] C_L^2 = f(F_h) C_L^2$$
(11)

The coefficient K_W must be selected to provide an approximation for the theoretical result over the Froude number range of interest. Wilson provides evaluations for the theory over a range of aspect ratios and depths which aid this approximation. It is of interest to note that Ref. 4 made K_W identical with the biplane factor to approximate the aspect ratio 10 and 0.084 depth/span result of Ref. 2 over the full Froude number range.

(13b)

Equation (11) can be written in the general form:

$$\frac{h/c}{K_W} C_{DW} = (q_D/q)^3 (2)^{q_D/q - 1} F_{h_D}^2 {}^{(q_D/q - 1)} \times \left[f \left(F_{h_D} \right) \right]^{q_D/q} C_{L_{VD}}^2$$
(12)

and in particular at the design speed, half-design dynamic pressure, and half-design speed points:

$$\frac{h/c}{K_W}C_{DW_D} = f(F_{h_D})C_{L_{VD}}^2 = fC_{L_{VD}}^2$$
 (13a)

$$\frac{h/c}{K_W} C_{DW_{qD/2}} \ = 16 F_{h_D}^2 \left[f \bigg(F_{h_D} \bigg) \right]^2 C_{L_{VD}}^2 \equiv 16 F^2 f^2 C_{L_{VD}}^2$$

$$\frac{h/c}{K_W}C_{DW_{VD/2}} = 512F_{h_D}^6 \left[f\left(F_{h_D}\right) \right]^4 C_{L_{VD}}^2 \equiv 512F^6 f^4 C_{L_{VD}}^2$$
(13c)

Therefore, by reference to Eq. (5) which was written for the same three speeds, the wave-drag coefficient may be written:

$$C_{DW} = k_{0W} + k_{1W}C_L + k_{2W}C_L^2$$

where

$$k_{0W} = \left(\frac{8}{3}f - 32F^2f^2 + \frac{512}{3}F^6f^4\right) \frac{K_W}{h/c} C_{LVD}^2$$

$$k_{1W} = \left(-2f + 40F^2f^2 - 256F^6f^4\right) \frac{K_W}{h/c} C_{LVD}$$

$$k_{2W} = \left(\frac{1}{3}f - 8F^2f^2 + \frac{256}{3}F^6f^4\right) \frac{K_W}{h/c}$$
(14)

Center-of-Gravity Effect

The forward and aft lift coefficients are related to the craft lift coefficient by:

$$C_{L_I} = \frac{l_2/l}{S_l/S} C_L \tag{15a}$$

$$C_{L_2} = \frac{l_1/l}{S_2/S} C_L$$
 (15b)

where l_1 and l_2 locate the center of gravity with reference to the forward and aft foil centers of pressure. The total drag due to lift is given by:

$$C_{DL} = \frac{S_{I}}{S} k_{0LI} + \frac{S_{I}}{S} k_{ILI} \frac{l_{2}/l}{S_{I}/S} C_{L} + \frac{S_{I}}{S} k_{2LI} \left(\frac{l_{2}/l}{S_{I}/S}\right)^{2} C_{L}^{2}$$

$$+ \frac{S_{2}}{S} k_{0L2} + \frac{S_{2}}{S} k_{IL2} \frac{l_{I}/l}{S_{2}/S} C_{L} + \frac{S_{2}}{S} k_{2L2} \left(\frac{l_{I}/l}{S_{2}/S}\right)^{2} C_{L}^{2}$$

$$= \frac{S_{I}}{S} k_{0LI} + \frac{S_{2}}{S} k_{0L2} + \left(\frac{l_{2}}{l} k_{ILI} + \frac{l_{I}}{l} k_{IL2}\right) C_{L}$$

$$+ \left[\frac{(l_{2}/l)^{2}}{S_{I}/S} k_{2LI} + \frac{(l_{I}/l)^{2}}{S_{2}/S} k_{2L2}\right] C_{L}^{2}$$
(16)

which provides the averaged drag-due-to-lift coefficients for the craft drag polar.

Relationship to Craft Performance

An illustrative drag polar is compiled in Tables 1 and 2. Within the limits of the variation of the propulsion coefficient with speed, the main performance characteristics of interest are proportional to one of three C_L/C_D ratios:

$$D = L/(C_L/C_D) \tag{17}$$

$$R_S = \frac{\text{P.C.}}{\text{sfc}} \frac{1}{L} \frac{C_L}{C_D}$$
 for a fixed sfc (18)

$$R_{S} = \left(\frac{\rho}{2}\right)^{-1/6} \frac{\text{P.C.}^{2/3}}{N^{1/3} C_{\text{sfc}}} \frac{I}{S^{1/6} \sqrt{L}} \frac{\sqrt{C_{L}}}{C_{D}^{2/3}}$$
(19)

where sfc = $C_{\rm sfc}/P^{1/3}$

$$P = \left(\frac{\rho}{2}\right)^{-1/2} \frac{L^{3/2}}{\text{P.C.}\sqrt{S}} / \frac{C_L^{3/2}}{C_D}$$
 (20)

$$E_{S} = \frac{(\rho/2)^{1/3}}{N^{1/3}} \frac{\text{P.C.}^{2/3}}{C_{\text{sfc}}} \frac{S^{1/3}}{L} \left(\frac{C_{L}^{3/2}}{C_{D}}\right)^{2/3}$$
(21)

The expressions for the maximum of each of these ratios and the corresponding lift coefficients, which identify the craft speed, are therefore of significance and are summarized in Table 3. For example, the two drag polars of Table 2 present the parametric performance of Table 4.

Table 3 and Eqs. (17-21) summarize the craft performance without recourse to graphical or numerical aids. Plots of the drag polar and of the lift-drag coefficients are given in Figs. 1-6. These plots can be used to relate analytical and traditional performance estimates.

The illustrative drag polars of Table 2 are shown on Fig. 1 where the significance of the minimum drag coefficient is obvious. The C_{L_D} is the design speed lift coefficient for the design weight, the maximum net weight afforded by the installed propulsion for specified takeoff and design speed-thrust margins. Reductions in the weight, as for fuel burn-off, reduce the design speed lift coefficient proportionately.

The drag polar sensitivity to weight on Fig. 1 is a complicating hydrodynamic distinctive introduced by the wave drag. Its effect is considered further in consideration of the specific range curves of Fig. 4.

The minimum power lift coefficient is significant because it is the maximum power-stabilized lift coefficient. Lift coefficients to four times the design speed lift coefficient are required but not normally for steady-state craft performance. Thus, drag estimate precision for the cruise drag polar is most important for the lift coefficient range from the minimum anticipated at design speed to the minimum power lift coefficient and for the corresponding speed ranges.

The D/q polar of Fig. 2 relates the craft drag to the propulsion system. That C_{L_D} which makes D/q and T/qM

Table 1 Parasite drag coefficients (Total foil area, $S = 14.62 \text{ m}^2$)

	$1000 C_{D_P} = 1000 D/qS$					
Forward foils Aft foil Forward pods Aft pod Forward struts Aft strut Total spray	3.889 1.804 1.313 1.307 0.509 0.486 0.964	+ 0.1554 + 0.0721 + 0.0479 + 0.0460 + 0.0201 + 0.0186	$C_L/C_{LVD} \\ C_L/C_{LVD} \\$			
Air Total	$\frac{0.865}{11.137}$	+0.3601	$C_L/C_{L_{VD}}$			

Note: Surface roughness and interference allowance are included in component drags.

Table 2 Drag polar coefficients (Total foil area, S = 14.62 m 2 , C_{L_D} = 0.2, nominal displacement = 100 mT, 1000 C_D = 1000 D/qS)

Parasite drag, C_{D_P}	11.14	$+\frac{0.3601}{C_{L_{VD}}}C_{L}$					
Separation drag, $C_{D_{\mathrm{sep}}}$	3.64	$-27.0 C_L$	$+50.0 C_L^2$				
Induced drag, $C_{D_i}^{-\text{sep}}$			$57.9 C_L^2$				
Surface image drag, $C_{D_{\rm surf}}$			$23.4 C_L^2$				
Wave drag, C_{DW}	$50.82 C_{L_{V\!D}}^2$	$-90.46C_{L_{\it VD}}C_{L}$	$+47.1 C_L^2$				
Total drag, $C_D \times 10^3 = 14.78 + 50.82 \ C_{L_{VD}}^2 + \left(\frac{0.3601}{C_{L_{VD}}} - 27.0 - 90.46 \ C_{L_{VD}}\right) C_L + 178.4 \ C_L^2$							
At design displacement, C_1 At 0.6 design displacement	$C_{VD} = C_{L_D}$: $C_D = 0.016$, $C_{L_{VD}} = 0.12$: $C_D = 0.00$	$82 - 0.04329 C_L + 0.1784 C_L$ $01551 - 0.03485 C_L + 0.1784$	C_L^2 .				

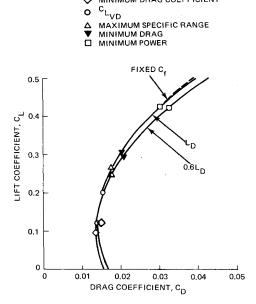
Table 3 Drag polar characteristics

$C_{D_{\min}}$	C_L	$-\frac{1}{2}\cdot\frac{k_1}{k_2}$			
Minimum drag coefficient	$C_{D_{\min}}$ $k_0 - \frac{l}{4} \frac{k_1^2}{k_2}$ Note alternative drag polar form: $C_D - C_{D_{\min}} = k_2 (C_L - C_{L_{CD_{\min}}})^2$				
$(C_L^{1/2}/C_D^{2/3})_{ m max}$ Max specific range for constant P.C. and sfc proportional to $P^{-1/3}$	C_L $(C_L^{1/2}/C_D^{2/3})_{ m max}$	$-\frac{1}{10}\frac{k_1}{k_2}\left(\sqrt{1+60\frac{k_0k_2}{k_1^2}}+1\right)$ Substitute C_L above into drag polar			
$(C_L/C_D)_{ m max}$ Minimum drag Max specific range for constant P.C. and sfc	C_L $(D/L)_{ m min}$	$\frac{\sqrt{k_0/k_2}}{2\sqrt{k_0k_2} + k_1}$			
$(C_L^{3/2}/C_D)_{\rm max}$ Min power Min flight speed Max specific endurance (all for constant P.C.)	C_L $(C_L^{3/2}/C_D)_{\text{max}}$	$-\frac{1}{2} \frac{k_1}{k_2} \left(\sqrt{l + 12 \frac{k_0 k_2}{k_1^2}} - I \right)$ Substitute C_L above into drag polar			

Note: For any $C_{L_{\text{ref}}}$: $V_{\text{ref}} / V_D = \sqrt{C_{L_{VD}} / C_{L_{\text{ref}}}}$.

Table 4 Illustrative performance summary $S = 14.62 \text{ m}^2$, $C_{L_D} = 0.2$, nominal displacement = 1 MN (100mT)

Characteristic	C_L	design net weight, V/V_D	L_D Value	C_L	$ At \ 0.6 \ L_D \\ V/V_D $	Value
C_{n} .	0.1213	1.284	0.01419	0.0977	1.108	0.01381
$C_{D_{\min}}$ $C_{L_{VD}}$	$C_{L_D} = 0.2$	1		0.12	1	***
$(C_L^{1/2}/C_D^{2/3})_{\rm max}$	0.2633	0.872	7.531	0.2488	0.694	7.293
$(L/D)_{\text{max}}$	0.3070	0.807	15.09	0.2949	0.638	14.21
$(C_L^{3/2}/C_D)_{\text{max}}$	0.4241	0.720	9.041	0.4223	0.533	8.414



MINIMUM DRAG COEFFICIENT

 \Diamond

0

Fig. 1 Drag polar.

MINIMUM DRAG COEFFICIENT o CLVD △ MAXIMUM SPECIFIC RANGE
▼ MINIMUM DRAG
□ MINIMUM POWER

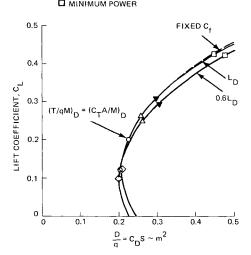
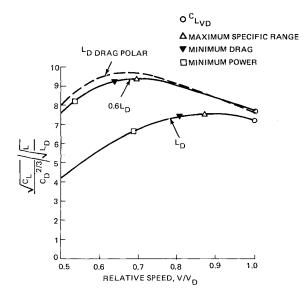


Fig. 2 D/q polar.

- MINIMUM DRAG COEFFICIENT
 O CLVD
- FIXED C, 6 5 C_D 2/3 0 0.2 0.3 0.1 0.4 0.5 LIFT COEFFICIENT, C

Fig. 3 Specific range curve.



115

Fig. 4 Specific range vs speed.

 $\diamondsuit \ \, \underset{\mathsf{C}}{\mathsf{MINIMUM}} \ \, \mathsf{DRAG} \ \, \mathsf{COEFFICIENT} \\ \bullet \ \, \mathsf{C}_{\mathsf{L}_{\mathsf{VD}}}$ △ MAXIMUM SPECIFIC RANGE
▼ MINIMUM DRAG
□ MINIMUM POWER

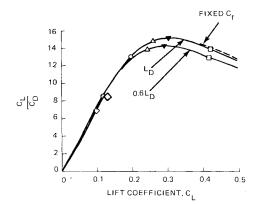


Fig. 5 Drag curve.

MINIMUM DRAG COEFFICIENT o^{c} L_{VD} Δ MAXIMUM SPECIFIC RANGE ▼MINIMUM DRAG ☐ MINIMUM POWER FIXED C, 0.6L_D 0.2 0.3 LIFT COEFFICIENT, C

Fig. 6 Power required curve.

identical for any specified thrust margin M corresponds to the maximum net weight available to the craft-propulsion system in the cruise mode. Partial throttle propulsion-system performance charts are entered through the relationship:

$$C_T = \frac{S}{A} C_D$$

The $C_L^{1/2}/C_D^{2/3}$ curves of Fig. 3 are parametric forms of the specific range curve. Traditional specific range-speed plots present various segments of these curves appropriate to the craft weight at various fuel weights, expressing the abscissa as speed and with appropriate scale factors on the ordinate.

Figure 4 presents another form of the specific range curves which displays the effect of weight and speed. The design weight curve and dashed 60% design weight curve of Fig. 4 are both transformations of the design weight curve of Fig. 3. Use of the design weight wave drag curve fit for both weight conditions produces the effect of high Froude numbers for the lightweight condition, reducing the wave drag and increasing the specific range. The error is not large and for some applications the adjustment of the wave drag component for craft weight produces a trivial precision improvement at the expense of intuitive appreciation for the more significant configuration-performance relationships.

Integrations of the specific range through the fuel burn-off depend upon the cruise mode employed. For the fixed lift coefficient cruise mode (Fig. 3) the integration is essentially at constant $C_L^{1/2}/C_D^{2/3}$ and fixed lift coefficient cruise at maximum $C_L^{1/2}/C_D^{2/3}$ presents maximum range. Fixed speed range (Fig. 4) becomes the integral of $\mathrm{d}C_L/C_D^{2/3}$ which cannot be expressed explicitly. Hydrofoil craft "range" and "endurance" are traditionally fixed speed range and range/speed as defined by the mid-fuel-weight drag polar.

The C_L/C_D curve of Fig. 5 serves in an identical way as a parametric drag curve. The L/D curve is a specific range curve for a fixed specific fuel consumption. Accountability for specific fuel consumption variability has only a slight effect upon the maximum specific range but significantly increases the speed for maximum specific range.

Without the identification for range the L/D curve has little practical significance. The maximum L/D is a form of weighted measure of the parasite and drag-due-to-lift minimizations but does not appear directly in any of the performance characteristics.

The $C_L^{3/2}/C_D$ curve of Fig. 6 is a parametric form of the power required or specific endurance curve. For a fixed propeller efficiency the lift coefficient for minimum power measures the minimum flight speed; variability of that efficiency usually increases the minimum flight speed. It is evident from the expression for the lift coefficient for minimum power in Table 3 that the minimum flight speed is reduced by increasing the drag polar coefficient k_0 or by reducing the drag polar coefficient k_2 . Increasing the foil depth and wetted strut area will accomplish this objective on some designs with only a small increase in minimum power because of the reduction in free surface drag.

Conclusion

The cruise performance characteristics for a given craft design at a given depth are fully defined parametrically by the three numbers which are the coefficients for the cruise drag polar. The precision with which those characteristics are defined is subject to the precision with which the individual

component drag coefficients are estimated and that is not the subject of this paper which is limited to consideration of the compilation of the drag polar coefficients and their application.

The nondimensional character of the drag polar distinguishes the performance contribution of the hydrodynamic configuration from that of the propulsion system and of dimensional factors such as design speed, power, and weight, all of which appear explicitly or implicitly as scale factors on the ordinates of Figs. 3-6. On the non-dimensional drag polar level hydrofoil craft of all sizes and speeds are found remarkably similar and subject to characterization as a class.

The product of the drag coefficient and foil area D/q at design speed is not determined by the craft configuration but by the propulsion which brings that product to the craft as T/q. The craft of given foil area is limited to that lift coefficient and C_LS , which is L/q, which matches the thrust. Thus, there may be some other foil area which would maximize the net weight available to the given T/q. This possibility introduces the requirement for a "general" (in S) drag polar which compiles the drag polar coefficients without assigning a value to S in order to determine that combination of lift coefficient and foil area which maximizes the net weight.

It is then possible that the foil area which maximizes net weight at design speed will not provide a sufficient thrust margin for the T/q at takeoff. For this two-point power-limited case, the design speed lift coefficient and foil area which maximize net weight are the solutions for design speed and takeoff general drag polars set equal to their respective T/q's.

Thus the particular (in S) drag polars of this paper are particular cases of general (in S) drag polars which can be employed to select a foil area. The particular drag polar is suited to the definition of craft performance in detail and with good precision. The general drag polar is suited to parametric characterization of configuration potential, ranging over power or speed under controlled conditions but at some expense in precision. It is the contribution of the drag polar to the precise definition and comprehension of such generalized design guidance which is significant rather than its application to a particular craft/propulsion case.

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